Answer ALL questions. Write your answers in the spaces provided.

1. The random variable A is normally distributed with a mean of 4 and a standard deviation of 3

The random variable B is independent of A and is normally distributed with a mean of 3 and a variance of 10

A random value is to be taken from each distribution.

(a) Find the probability that the value of A is more than twice the value of B.

(4)

The random variable T is given by

$$T = A_1 + A_2 + \cdots + A_n + nB$$

where $A_1, A_2, \ldots A_n$ and B are independent.

(b) Given that T has a variance of 196, find the value of n.

(4)

Question 1 continued

(Total for Question 1 is 8 marks)

2. In a car factory there are two production lines, A and B. A random selection of cars is taken from each production line and the maximum power output, x, of each car is recorded.

The results for the cars from production line A are

92.5 94.8 89.9 88.3 91.4 92.0 93.7 91.9 93.0

The results for the cars from production line B are summarised by

$$n_B = 13 \qquad \sum x_B = 1175 \qquad \sum x_B^2 = 106\,800$$

The maximum power outputs of the cars from each production line are normally distributed.

(a) Test, at the 2% level of significance, whether or not there is evidence of a difference in the variances of the maximum power outputs of the cars from the two production lines. You should state your hypotheses and critical value.

(7)

The factory manager is concerned that the maximum power outputs of cars from production line B are too inconsistent. Factory guidelines state that the standard deviation of the maximum power output of cars from a production line should not be greater than 3.5

(b) Test, at the 5% level of significance, whether or not there is evidence that the standard deviation of the maximum power output of cars from production line B is too high. You should state your hypotheses and critical value and show your working clearly.

(4)

Question 2 continued

(Total for Question 2 is 11 marks)

3. A village show has a jam making competition. The 8 jams submitted for the competition are tasted and each is awarded points by each of the three judges. Some of the points awarded are shown in Table 1

Jam	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*
Judge 1	15	15	29	13	10	25	9	16
Judge 2	10	16	17	12	13	15	9	23
Judge 3		16	18	12		19	13	15

Table 1

(a) Calculate Spearman's rank correlation coefficient between the points awarded by Judge 1 and the points awarded by Judge 2

(5)

(b) Explain why Spearman's rank correlation coefficient is used in this case.

(1)

Judge 3 had no tied ranks and only recorded his 6 highest points, which are shown in Table 1

Given that Spearman's rank correlation coefficient between the points awarded by Judge 2 and the points awarded by Judge 3 is exactly $\frac{4}{7}$

(c) explain which of jam *A* and jam *E* was awarded the lowest points from Judge 3. Show your working clearly.

(5)

Question 3 continued

(Total for Question 3 is 11 marks)

4. The continuous random variable X can only take values in the interval [0, 1]

After taking a series of observations of X, Adam suggests that the following probability density function might be a suitable model for X

$$f(x) = \begin{cases} kx(1 - x^2) & 0 \leqslant x \leqslant 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Use algebraic integration to find the value of the constant k.

(4)

(b) Showing your working, find the mode of X using Adam's model.

(2)

Beatrice takes an independent sample of observations of X and suggests that a more suitable model might be defined by the cumulative distribution function

$$H(x) = \begin{cases} 0 & x < 0 \\ \dfrac{\ln(1 + x)}{\alpha} & 0 \leqslant x \leqslant 1 \\ 1 & x > 1 \end{cases}$$

(c) Show that the constant $\alpha = \ln 2$

(2)

(d) Using calculus and showing your working, find the exact value of the mean of X using Beatrice's model.

(7)

(e) Giving reasons for your answers, compare the skewness of the distributions suggested by Adam and Beatrice.

(2)

Question 4 continued

Question 4 continued

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Question 4 continued

(Total for Question 4 is 17 marks)

5. Two independent random variables X and Y have distributions $X \sim \mathrm{N}(\mu_x, 3^2)$ and $Y \sim \mathrm{N}(\mu_y, 4^2)$

A random sample of size n from X has mean $\overline{x} = 17$

A random sample of size $2n$ from Y has mean $\overline{y} = 23$

A test is carried out at the 5% significance level using the hypotheses

$$\mathrm{H}_0 : \mu_y - \mu_x = 4 \qquad \mathrm{H}_1 : \mu_y - \mu_x > 4$$

Find the range of values of n for which H_0 is rejected.

(6)

Question 5 continued

(Total for Question 5 is 6 marks)

6. The random variables X and Y each have normal distributions with the same variance.

A researcher takes random samples from each distribution and calculates the following summary statistics,

$$\bar{x} = 76.5 \qquad s_x^2 = 24$$

$$\bar{y} = 81.2 \qquad s_y^2 = 27$$

Given that $n_y = n_x + 4$ and that the pooled estimate of variance, $s_p^2 = 25.8$

(a) show that $n_x = 9$

(4)

The researcher believes that the population mean of Y, μ_y is greater than 2 more than the population mean of X, μ_x

(b) Test whether or not the data provides evidence to support the researcher's belief at the 10% significance level. Show your working and state your hypotheses clearly.

(5)

Question 6 continued

Question 6 continued

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Question 6 continued

(Total for Question 6 is 9 marks)

7. A shop supplies balloons for children's parties. Following complaints the manager tests the balloons to see at what pressures they burst. A random sample of 7 balloons is tested and the pressures at which they burst (x Pascals above atmospheric pressure) are given below

1734 1759 1777 1723 1747 1699 1740

The manager decides to use a t-distribution to calculate a 90% confidence interval for the mean pressure at which balloons burst.

(a) State a necessary assumption that he needs to make in order for this calculation to be valid.

(1)

(b) Showing your working, use the data given above to calculate this confidence interval, giving your answers correct to 4 significant figures.

(5)

The shop manager visits the balloon manufacturer. He is told that the standard deviation of the pressures at which the balloons burst is known to be 22 Pascals.

He is shown the results of an experiment in which 40 balloons were tested and produced a k % confidence interval for the mean pressure at which balloons burst of

$$1728.91 < \mu < 1741.09$$

(c) Showing your working, calculate the value of k.

(5)

The manufacturer repeated this experiment to produce 5 independent k % confidence intervals for the mean, each time using samples of size $n = 40$

(d) Find the probability that all 5 of these confidence intervals contain the true value of the population mean.

(2)

Question 7 continued

Question 7 continued

(Total for Question 7 is 13 marks)

TOTAL FOR PAPER IS 75 MARKS